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LETTER TO THE EDITOR

Non-linear electron plasma waves

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Abstract. It is shown that a steady state non-linear electron plasma wave can have saw-tooth shaped structure.

Propagation of an electron plasma wave in an unmagnetised plasma is well understood. Recently, Yu (1976) has shown that a non-linear electron plasma wave can propagate as a solitary pulse. The latter moves with a speed close to the electron thermal speed.

In this letter, we show that the non-linearity associated with the electron inertial motion gives rise to the non-linear steepening of the electron plasma wave. As a result, we expect the non-linear electron plasma wave to have a saw-tooth shaped profile.

Consider the propagation of an electron plasma wave in an unmagnetised plasma. Since we shall be concerned with high-frequency wave phenomena, ions are assumed to form a neutralising background. The wave dynamics is then governed by

$$n_t + (nv)_x = 0, \tag{1}$$

$$v_t + vv_x = \phi_x - (\ln n)_x, \tag{2}$$

$$\phi_{xx} = (n-1), \tag{3}$$

where the electron density n is normalised by the average particle number density n_0, the fluid velocity v by $v_e = (T_e/m)^{1/2}$ the electron thermal velocity, t by ω_{pe}^{-1}, ϕ by T_e/e, and the length x by $\lambda_e = v_e/\omega_{pe}$ the Debye length ($\omega_{pe}^2 = 4\pi n_0 e^2/m$). Other notation is standard.

If the system exhibits a propagating stationary wave, it is rather convenient to express all the physical quantities in a single variable, namely,

$$\xi = x - Ut, \tag{4}$$

where $U = V/v_e$, and V is the velocity of the non-linear wave in the moving frame.

Letting $n = 1 + N$, we get from (1)

$$v = UN/(1+N), \tag{5}$$

where $N = \tilde{n}/n_0$.

Differentiating (2) with respect to x, and using (3) we find

$$v_{xt} + (v_x)^2 + vv_{xx} = (n-1) - (\ln n)_{xx}. \tag{6}$$

From (4), in the steady state, we have $\partial_t = -U\partial_\xi$, and $\partial_x = \partial_\xi$. hence, (6) becomes

$$(v-U)v_{\xi\xi} + (v_\xi)^2 = N - [\ln(1+N)]_{\xi\xi}. \tag{7}$$

Substituting (5) into (7) we obtain after some simple algebra

$$\left(\frac{U^2}{(1+N)^3} - \frac{1}{1+N}\right)N_{\xi\xi} + \left(\frac{1}{(1+N)^2} - \frac{3U^2}{(1+N)^4}\right)(N_\xi)^2 + N = 0. \tag{8}$$

This equation governs the propagation of a non-linear electron plasma wave. In general, to obtain the solution of (10) is a formidable task. However, to have some physical insight into the problem under consideration, we limit ourselves to small amplitudes. Correspondingly, equation (8) yields

$$\alpha N_{\xi\xi} - \beta(N_\xi)^2 + N = 0, \tag{9}$$

where $\alpha = U^2 - 1$, and $\beta = 3U^2 - 1$.

Introducing $H = (2\beta/\alpha)N$, and $\zeta = (2/\alpha)^{1/2}\xi$, we rewrite (9) in the form

$$2H_{\zeta\zeta} - (H_\zeta)^2 + H = 0. \tag{10}$$

It follows that for $U^2 > 1$, we have $H > 1$. Thus, equation (10) admits saw-tooth shaped solutions (Alterkop and Rukhadze 1972). The period of the oscillation is

$$T = 4[G_m + (2\ln G_m)^{1/2}/G_m + \ldots], \tag{11}$$

where $G_m^2 = H_m$. Physically, the saw-tooth shaped profile occurs due to the excitation of a large number of different harmonics, so that the saw-tooth has a non-harmonic character. A similar result is also encountered for finite amplitude ion–cyclotron waves across the external magnetic field (Chaturvedi 1976).

In summary, we have shown that non-linear electron plasma waves will have saw-tooth shaped profiles. It is worth mentioning that our analysis does not take into account the interaction of an electron plasma wave with slow plasma motion. The phenomena of oscillating-two-stream instability and the envelope Langmuir solitons (Zakharov 1972) usually occur on ω_{pi}^{-1} time scale, where ω_{pi} is the ion plasma frequency. Finite amplitude Langmuir solitons are discussed elsewhere (Schamel *et al* 1977).

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